TORSIONAL VIBRATION OF SYMMETRICAL STRUCTURES

BY

W. K. Tso [I] and K. G. Asmis [II]

SYNOPSIS

The possibility of induced torsional motions to a symmetrical structure subjected to ground motion is illustrated. The induced torsional motions are caused by the nonlinear coupling between the lateral and torsional motions. This coupling arises due to the nonlinear force-displacement relations of the resisting elements of the structure. Examples based on two types of force-displacement relationships are given to show how the torsional motions are induced. The implication of such torsional motion on the ductility requirement of the resisting elements located at the periphery of the building is discussed.

GLOSSARY OF TERMS

$G_{x,y}$	=	ground acceleration
I_p	=	mass polar moment of inertia
M	=	total mass of platform
$R_{x,y}$	=	resisting elements
a,b	=	plan dimensions of structure as shown in Fig. 1
k_x	=	stiffness constant
n	=	constant described in equation 17
u,v	=	horizontal lateral displacement
α	=	constant described in equation 17
δ_j	=	displacement of resisting elements
ε	=	measure of nonlinearity defined in equation 15
θ	=	torsional rotation
η	=	frequency ratio ω_x/ω_θ

[I] Associate Professor, Department of Civil Engineering and Engineering Mechanics, McMaster University, Hamilton, Ontario, Canada.

[II] Graduate Student, Department of Civil Engineering and Engineering Mechanics, McMaster University, Hamilton, Ontario, Canada.

τ = non-dimensional time

$\Phi_{x,\theta}$ = restoring force functions

ω_x = linear natural frequency of translation

ω_θ = linear natural frequency of rotation

INTRODUCTION

Torsional motions of a structure caused by earthquakes may lead to severe damages. The causes of torsional motion are generally ascribed to asymmetrics of the building. The asymmetry may be due to the uneven distribution of stiffnesses, and/or uneven distribution of masses of the structure. Therefore, the code requirements for torsional resistance are based mainly on the amount of eccentricity of the structure [1, 2].

Recently, it was pointed out that symmetrical buildings may be subjected to torsional motion due to the passage of seismic wave motion [3]. The torsional motion is induced on the symmetrical structure if the phases of the seismic wave at different points of the ground on which the structure is built are different. Therefore, symmetrical buildings with plan dimensions comparable to the wave length of the seismic wave would be particularly susceptible to torsional motions due to this cause.

In the present paper, another mechanism through which torsional motions may be induced to symmetrical structures is discussed. This mechanism arises due to the nonlinear coupling between the lateral translational motions and the torsional motions. This coupling is caused by the nonlinear force-displacement characteristics of the restoring forces of the structure. While lateral motions are caused by the horizontal ground motion, the torsional motions are parametrically excited by the lateral motions. The condition of exciting torsional motions by this mechanism depends mainly on the ratio of the lateral and torsional frequencies of the structure.

STATEMENT OF PROBLEM

Let us consider a structure consisting of a stiff rectangular platform with a total mass M and a mass polar moment of inertia I_p. The elements of lateral resistance are taken to be distributed along the perimeter of the structure, as shown in Fig. 1. This system has three degrees of freedom in movement, namely, the horizontal lateral displacements u and v in the two principal directions and rotation θ about a vertical axis. The equations of motion can be obtained by considering the general displaced position of the platform, by an amount u in the X direction, an amount v in the Y direction and rotated through an angle θ as shown in Fig. 2. Taking the angle of rotation θ to be small and assuming that the stiffness of the resisting elements out of its plane is negligible, the equations of motion can be written as

$$M \ddot{u} + R_x(\delta_1) + R_x(\delta_2) = -M G_x \tag{1}$$

$$M \ddot{v} + R_y(\delta_3) + R_y(\delta_4) = -M G_y \tag{2}$$

$$I_p \ddot{\theta} + bR_x(\delta_2) - bR_x(\delta_1) + aR_y(\delta_4) - aR_y(\delta_3) = 0 \tag{3}$$

where δ_j ($j = 1,2,3,4$) is the displacement of the lateral resisting element j in the direction of the plane of the element j. Dots represent differentiation with respect to time t and G_x and G_y are the ground acceleration in the x and y directions respectively. The force-displacement relationship is assumed to be anti-symmetric, namely

$$R(\delta) = -R(-\delta) \tag{4}$$

From Fig. 2, δ_j can be expressed in terms of the displacements of the center of mass by

$$\delta_1 = u - b\theta \tag{5}$$

$$\delta_2 = u + b\theta \tag{6}$$

$$\delta_3 = v - a\theta \tag{7}$$

$$\delta_4 = v + a\theta \tag{8}$$

If the ground motion is in the X direction only, the lateral movement v in the y direction will not be excited. Therefore, only equations (1) and (3) need to be considered in this case. In order to allow for a variety of restoring force-displacement relationships, it is most convenient to rewrite equations (1) and (3) in the form

$$\ddot{u} + \omega_x^2 \, \Phi_x(u,\theta) = -G_x \tag{9}$$

$$\ddot{\theta} + \omega_\theta^2 \, \Phi_\theta(u,\theta) = 0 \tag{10}$$

$\Phi_x(u,\theta)$ is proportional to the restoring force in the X direction when the structure is displaced an amount u and rotated through an amount θ. Similarly, $\Phi_\theta(u,\theta)$ is proportional to the restoring torque when the structure is displaced from its equilibrium configuration. ω_x and ω_θ represent the linear lateral and torsional natural frequencies respectively. They can be calculated by the initial slope of the force-displacement relationship. The formulae for determination of ω_x and ω_θ for a symmetrical building with resisting elements distributed in a variety of ways is given in ref. [3].

If the force-displacement relationship is linear such that

$$R_x(\delta) = k_x\delta \tag{11}$$

then $\Phi_x(u,\theta) = u$ and $\Phi_\theta(u,\theta) = \theta$. In this case, equations (9) and (10) are uncoupled. Equation (9) represents the single-degree-of-freedom oscillator and the torsional motion is not excited. However, if the force-displacement relationship is nonlinear, Φ_x and Φ_θ in general are complicated functions of both u and θ. Equations (9) and (10) are then coupled together and response in u may give rise to a torsional response θ. Since the lateral resisting elements are often loaded beyond their linear elastic limit during an earthquake, it is the rule rather than the exception that the force-displacement relationships in the resisting elements are nonlinear.

To facilitate computations, a nondimensional time variable τ is introduced. τ is defined by

$$\tau = \omega_\theta t \tag{12}$$

Equations (9, 10) can then be written as

$$\frac{d^2u}{d\tau^2} + 0.01\frac{du}{d\tau} + \eta^2\,\Phi_x(u,\theta) = \frac{-G_x}{\omega_\theta^{\ 2}} \qquad (13)$$

$$\frac{d^2\theta}{d\tau^2} + 0.01\frac{d\theta}{d\tau} + \Phi_\theta\,(u,\theta) = 0 \qquad (14)$$

Two small viscous damping terms have been added in equations (13) and (14) to represent the damping in the system both in translation and rotation The added damping amounts to approximately 1% critical damping for systems considered in the computations.

FORCE-DISPLACEMENT RELATION

The force-displacement relationship of the resisting elements differs depending on the form and material of construction of the elements. The resisting element may take the form of planar frames made of steel, concrete or timber. Also, it may take the form of shear walls made of concrete and reinforced masonry. In this paper, two types of force-displacement curves are considered.

Type I Curve:- The force-displacement relation below the yield level is taken to be elastic with a small softening nonlinearity. Once yielding starts, the element deforms in a plastic manner. Mathematically it may be written as

$$R\,(\delta) = k_x(\delta - \varepsilon\delta^3) \qquad (15)$$

and

$$|R\,(\delta)| < R\ \text{yield} \qquad (16)$$

ε is taken to be a small quantity. A plot of a Type I Curve is shown in Figure 3. It can be seen that the curve is a good representation of the force-displacement relationship for a wide class of lateral load resisting elements.

Type II Curve:- The force-displacement curve is taken to be hysteretic, as shown in Fig. 4. It consists of a backbone curve and one ascending and one decending branch which forms the hysteretic loop. This type of force displacement curve is representative of cross-braced frames and towers [4]. Such resisting elements do not take appreciable force when the structure is deflected in the opposite direction after yielding in tension in the other direction. Mathematically, they can be most conveniently described by the expressions

$$\frac{\delta}{\delta_y} = \frac{R}{R_y} + \alpha\left(\frac{R}{R_y}\right)^n \qquad (17)$$

and

$$\frac{\delta - \delta_0}{2\delta_y} = \frac{R - R_0}{2R_y} + \alpha\left(\frac{R - R_0}{2R_y}\right)^n \qquad (18)$$

where α is a constant and n is an odd integer. R_y and δ_y denote the yield force and displacement respectively and R_0 and δ_0 denote the most recent point in the force-displacement plot at which the loading has been reversed. Equation (17) describes the backbone curve and equation (18) describes the ascending and decending branches of the hysteresis loop. The curves are characterized by the fact that the initial stiffnesses on first loading, on unloading and on reloading are equal to each other. Force-displacement relations as shown in Fig. 4 are sometimes known as hysteretic slip models.

While transforming equations (15) and (16) or equations (17) and (18) into the form $\Phi_x(u,\theta)$ and $\Phi_\theta(u,\theta)$ may be complex analytically, one can generate the values of $\Phi_x(u,\theta)$ and $\Phi_\theta(u,\theta)$ using a computer, based on equations (15, 16) or equations (17, 18). Since equations (13) and (14) will be solved numerically by means of a computer, it is not necessary to express the force-displacement curve in an analytical form of $\Phi_x(u,\theta)$ and $\Phi_\theta(u,\theta)$.

INDUCED TORSIONAL RESPONSE

Equations (13, 14) are numerically integrated for three kinds of ground excitation. Shown in Fig. 5 is the response of the structure subjected to 13 cycles of sinusoidal ground excitation. The restoring elements of the structure are assumed to have a Type I force-displacement relation. The structure has a torsional period τ = 0.35 sec., and the frequency ratio η = 0.7. The period of the ground excitation is taken to be 0.5 sec. Fig. 5b shows the force-displacement relationship used in the calculation. Fig. 5c gives the lateral response u while Fig. 5d shows the induced torsional response θ. By comparing Fig. 5c and 5d, it can be seen that the torsional motion is excited only after the lateral response has reached a large magnitude. Shown in Fig. 5a is the displacement at the periphery of the structure where resisting element 1 is located. A comparison between Fig. 5a and Fig. 5c makes it evident that the displacement at the periphery of the structure is much larger than the displacement at the mass center.

If torsional motion is not excited, the displacements at the resisting element 1 would be the same as that of the center of the structure. The displacement at the center of the structure can be evaluated by treating the structure as a single degree of freedom system capable of movement in the X direction only. Therefore, the ductility requirements on the resisting elements at the periphery of the structure become exceptionally severe once torsional motion is excited, as demonstrated in Fig. 5.

Shown in Fig. 6 is the response of a similar system subjected to the 1940 El Centro earthquake ground acceleration record. The torsional period of the structure is taken to be 0.35 sec., and the lateral to torsional frequency ratio is 0.85. The resisting elements are assumed to have the same Type I force-displacement relationship as shown in Fig. 5b. Fig. 6a shows the 1940 El Centro earthquake ground acceleration record and Fig. 6c and Fig. 6d show the lateral and torsional responses of the structure. Fig. 6b shows the displacement at the resisting element 1. It can be seen that the maximum displacement at the periphery of the structure is more than twice the maximum lateral response u.

In Fig. 7, the response of the structure to the El Centro earthquake ground acceleration with a lateral to torsional frequency ratio of 0.6 is given. The resisting elements of the structure are assumed to have a Type II force-displacement curve as shown in Fig. 7b. A comparison of the lateral response u and torsional response θ as shown in Fig. 7c and Fig. 7d respectively shows clearly that torsional motion is excited only when the lateral response is large. Again, the displacement at the periphery is shown in Fig. 7a. In this case, the maximum displacement at the periphery is 1.6 times as large as the maximum lateral displacement u.

Finally, the same structure as used in Fig. 7 is subjected to the 1952 Taft earthquake ground acceleration excitation. The responses are shown in Fig. 8. Since the 1952 Taft earthquake acceleration record is less severe than the El Centro record, the lateral response u is not as large. Consequently, the torsional motion is only excited for a short period when the lateral response is substantial. Fig. 8b shows the combined effect of lateral and torsional response. The maximum response at the periphery is only 20% larger than the lateral response in this case.

DISCUSSION AND CONCLUSION

In Fig. 5 to Fig. 8, the responses of a symmetrical structure subjected to three different ground excitations are given. The resisting elements of the structure are assumed to be either of the nonlinear yielding or the hysteretic slip-type. It is shown that in all cases torsional motions are induced due to the nonlinear coupling with the lateral response.

The torsional motion is excited at instances when the lateral response of the structure is large. With the torsional motion excited and the lateral response large, the combined effect of translation and rotation of the structure places very severe requirements on the ductile behaviour of the resisting elements at the periphery of the structure. Ductility requirements based on the calculation of lateral response alone will be insufficient if torsional motions are also induced. As shown in the examples of the present paper, the ductility requirement on the resisting elements may be twice as large as that based on calculations of the lateral response alone. Therefore, from a design point of view, one should always take into account the possible torsional motions of the structure in estimating the ductility requirements of the resisting elements located at the periphery of the structure.

It should be pointed out that the lateral to torsional frequency ratio η is an important parameter to determine whether torsional response will be excited. Consider the case of the responses as shown in Fig. 6 where $\eta = 0.85$. If the frequency ratio is changed to 0.7, the torsional motion will not be excited. The critical range of frequency ratio in which torsional motions are likely to be excited depends both on the force-displacement characteristic of the resisting elements and the characteristic of the ground excitation. For the different combinations of resisting elements and ground excitations investigated, the critical range has a spread from 0.6 to 0.9. Further classification of the critical frequency range with respect to resisting element characteristics is needed in order to assess the likelihood of this type of parametrically excited torsional motion occurring in symmetrical structures.

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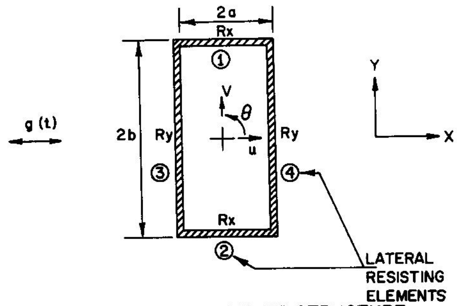

FIGURE.1 PLAN OF STRUCTURE

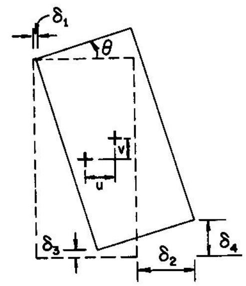

FIGURE.2 DISPLACED POSITION OF STRUCTURE

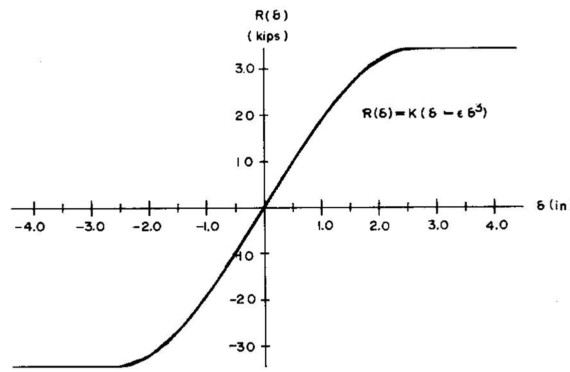

FIG. 3 CUBIC-NONLINEAR ELASTIC PLASTIC FORCE-DISPLACEMENT
(R_Y = 34.3 KIPS)

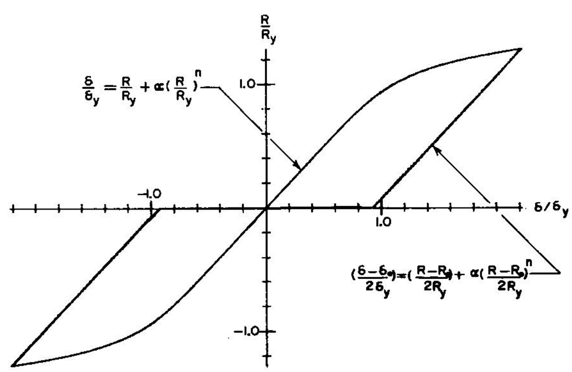

FIG. 4 HYSTERETIC SLIP MODEL FORCE-DISPLACEMENT CURVE
(α = 0.10; N = 9)

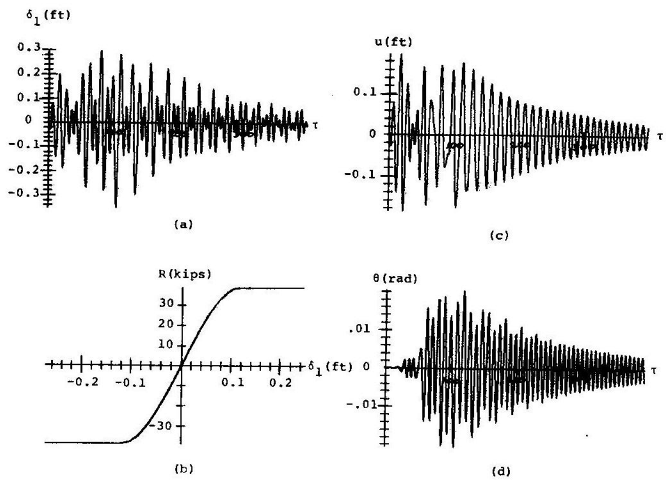

FIG. 5 RESPONSE OF SYSTEM TO 13 CYCLE SINUSOIDAL EXCITATION
(T_θ = 0.35 SEC; n = 0.7)

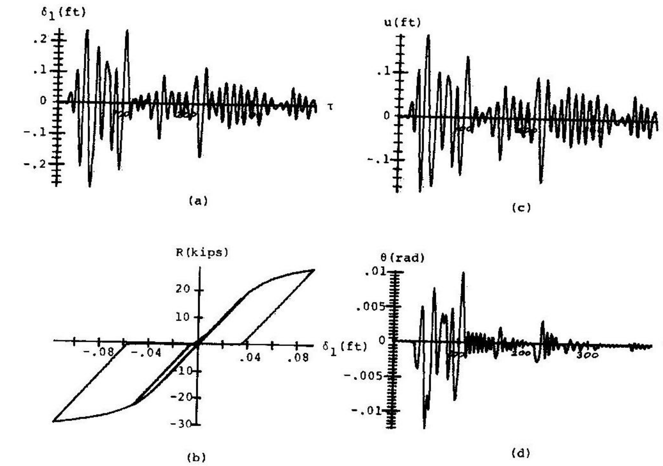

FIG. 7 RESPONSE OF SYSTEM TO EL CENTRO 1940 EXCITATION
(T_θ = 0.35 SEC; n = 0.6)

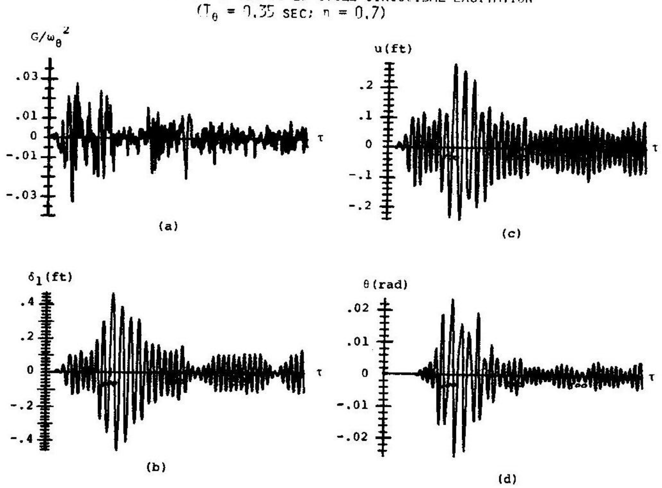

FIG. 6 RESPONSE OF SYSTEM TO EL CENTRO 1940 EXCITATION
(T_θ = 0.35 SEC; n = 0.85)

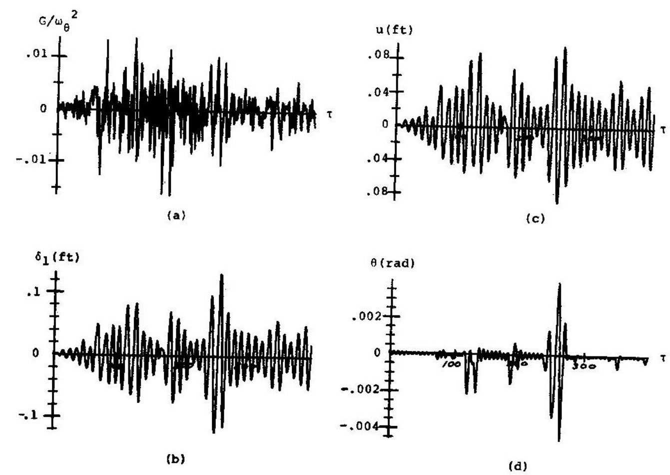

FIG. 8 RESPONSE OF SYSTEM TO TAFT 1952 EXCITATION
(T_θ = 0.35; n = 0.6)